Mechanics of Materials

For

ME/CE

By

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Syllabus for Mechanics of Materials

Stress and Strain, Elastic Constants, Poisson's Ratio; Mohr's Circle For Plane Stress and Plane Strain; Thin Cylinders; Shear Force and Bending Moment Diagrams; Bending and Shear Stresses; Deflection of Beams; torsion of Circular Shafts; Euler's Theory of Columns; Energy Methods; Thermal Stresses; Strain Gauges and Rosettes; Testing of Materials With Universal Testing Machine; Testing of Hardness and Impact Strength, Theories of Failures, Shear Centre.

Previous Year GATE Papers and Analysis

GATE Papers with answer key

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Contents

"It is easy to sit up and take notice. What is difficult is getting up and taking action."
....Al Batt

CHAPTER 1

Simple Stress and Strain

Learning Objectives

After reading this chapter, you will know:

1. Simple Stress and Strain, Stress Strain Diagram
2. Hooke's Law for Axial and Shearing Deformations
3. Poisson's Ratio for Biaxial and Triaxial Deformations
4. Cylindrical Pressure Vessel, Spherical Pressure Vessel

Introduction

Strength of materials deals with the elastic behavior of materials and the stability of members. The concept of strength of materials is used to determine the stress and deformation of axially loaded members, connections, torsion members, thin-walled pressure vessels, beams, eccentrically loaded members and columns. In this chapter we will study the stress and strain due to axial loading, temperature change and thin walled pressure vessels.

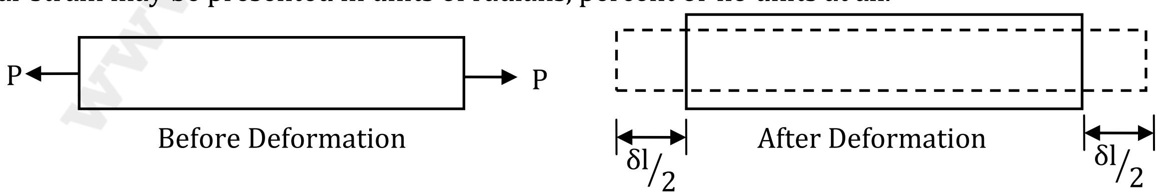
Simple Stress and Strain

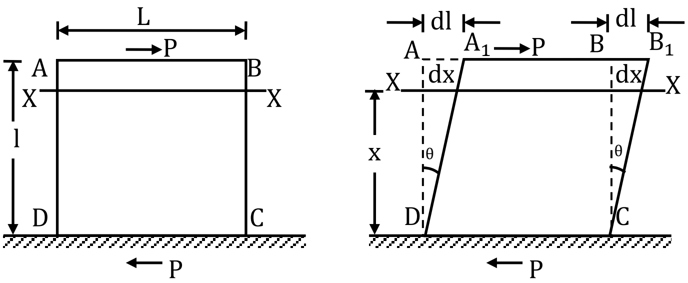
Stress (σ) is the internal resistance offered by the body per unit area to the applied force (σ). Typical units of stress are N/m^2, ksi and MPa. There are two primary types of stresses, normal stress and shear stress. Normal stress σ, is calculated when the force is normal to the surface area whereas the shear stress τ, is calculated when the force is parallel to the surface area. Mathematically stress is force per unit area.

$$\text{Normal Stress}(\sigma) = \frac{P_{\text{Normal to Area}}}{A}$$

$$\text{Shear Stress }(\tau) = \frac{P_{\text{Parallel to Area}}}{A}$$

Linear strain (normal strain, longitudinal strain, axial strain) ε, is a change in length per unit length. Linear strain has no units. Shear strain(γ) is an angular deformation resulting from shear stress. Shear strain may be presented in units of radians, percent or no units at all.





Before Deformation After Deformation

$$\varepsilon = \frac{\delta l}{L}$$

$$\gamma = \frac{\delta_{Parallel}}{Height} = \tan\theta \simeq \theta \text{ [Where } \theta \text{ is too small]}$$

Hooke's Law: Axial and Shearing Deformations

Hooke's law is a simple mathematical relationship between elastic stress and strain: Stress is proportional to strain. For normal stress, the constant of proportionality is the modulus of elasticity (Young's Modulus), E.

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

The deformation δ, of an axially loaded member of original length L can be derived from Hooke's law. Tension loading is considered to be positive, compressive loading is negative. The sign of the deformation will be the same as the sign of the loading.

$$\delta l = L\varepsilon = L\left(\frac{\sigma}{E}\right) = \frac{PL}{AE}$$

Where, $\frac{AE}{L}$ define as axial stiffness of bar

This expression for axial deformation assumes that the linear strain is proportional to the normal stress($\varepsilon = \sigma/E$) and that of the cross-sectional area is constant.

When an axial member has distinct sections differing in cross-sectional area or composition, superposition is used to calculate the total deformation as the sum of individual deformations.

$$\delta l = \sum \frac{PL}{AE} = P\sum \frac{L}{AE}$$

When one of the variables varies continuously along the length,

$$\delta l = \int \frac{PdL}{AE} = P\int \frac{dL}{AE}$$

The new length of the member including the deformation is given by:

$$L_f = L + \delta l$$

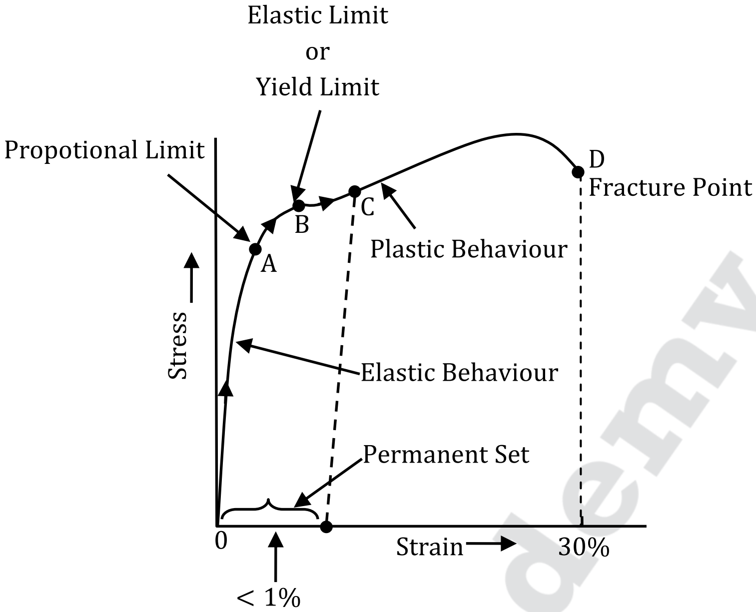
The algebraic deformation must be observed.

Hooke's law may also be applied to a plane element in pure shear. For such an element, the shear stress is linearly related to the shear strain by the shear modulus, also known as the modulus of rigidity (G)

The relationship between shearing deformation, δ_s and applied shearing force, P is then expressed by

$$\delta_s = \frac{PL}{AG}$$

Stress-Strain Diagram



Proportional Limit: It is the point on the stress strain curve up to which stress is proportional to strain.

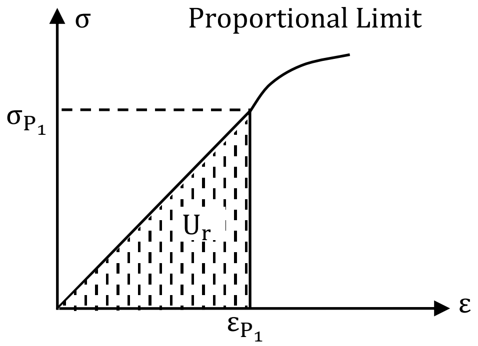
Elastic Limit: It is the point on the stress strain curve up to which material will return to its original shape when unloaded.

Yield Point: It is the point on the stress strain curve at which there is an appreciable elongation or yielding of the material without any corresponding increase of load indeed the load actually may decrease while the yielding occurs.

Ultimate Strength: It is the highest ordinate on the stress strain curve.

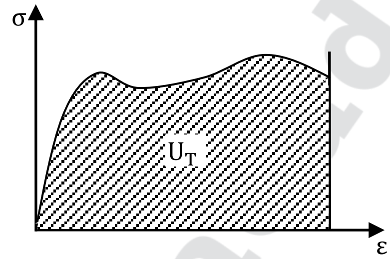
Rupture Strength: It is the stress at failure.

Modulus of Resilience (U_r): The work done on a unit volume of material, as a simple tensile force is gradually increased from zero to such a value that the proportional limit of the material is reached is defined as the modulus of resilience. This may be calculated as the area under the stress-strain curve from the origin up to the proportional limit.



$$U_r = \frac{1}{2}\sigma_{P_1} \times \varepsilon_{P_1} = \frac{1}{2}\frac{\sigma_{pl}{}^2}{E}$$

Modulus of Toughness (U_T): The work done on a unit volume of material as a simple tensile force is gradually increased from zero to the value causing rupture is defined as the modulus of toughness. This may be calculated as the entire area under the stress-strain curve from the origin to rupture. Toughness of a material is its ability to absorb energy in the plastic range of the material.



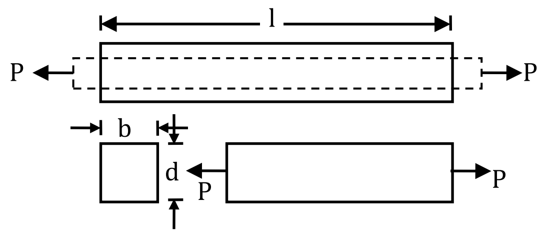
Percentage Elongation: The increase in length of a bar after deformation divided by the initial length and multiplied by 100 is the percentage elongation. Both the percentage reduction in area and the percentage elongation are considered to be measures of the ductility of a material.

$$\text{Percentage Elongation} = \frac{\Delta L}{L} \times 100$$

Strain Hardening: If a ductile material can be stressed considerably beyond the yield point without failure, it is said to strain-harden. This is true of many structural metals.

Poisson's Ratio Biaxial and Triaxial Deformations

Poisson's ratio ν, is a constant that relates the lateral strain to the longitudinal strain for axially loaded members.



The ratio $\frac{\delta l}{l}$ is called "**Longitudinal Strain**" while $\frac{\delta b}{b}$ or $\frac{\delta d}{d}$ is called "**Lateral Strain**"

$$\nu = \frac{\text{Lateral strain}}{\text{Longitudinal Strain}} = \frac{1}{m} = \text{Poisson's Ratio}$$

Note: ν varies from 0 to 0.5; $\nu = 0.5$ for incompressible materials and $\nu < 0.5$ for viscous materials.
ν for soft materials $\Rightarrow$ 0.25 to 0.5; ν for hard materials $\Rightarrow$ 0 to 0.5

Theoretically, Poisson's ratio could vary from 0 to 0.5, but typical values are 0.33 for aluminum and 0.3 for steel and maximum value of 0.5 for rubber. Poisson's ratio is zero for cork.

Poisson's ratio permits us to extend Hooke's law of uniaxial stress to the case of biaxial stress. Thus if an element is subjected simultaneously to tensile stresses in x and y direction, the strain in the x direction due to tensile stress, σ_x is σ_x/E. Simultaneously the tensile stress, σ_y will produce lateral contraction in the y direction of the amount $\nu \times (\sigma_y/E)$, so the resultant unit deformation or strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

Similarly, the total strain in the y direction is

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$$

Hooke's law can be further extended for three-dimensional stress-strain relationships and written in terms of the three elastic constants, E, G and ν. The following equations can be used to find the strains caused due to simultaneous action of triaxial tensile stresses.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

For an elastic isotropic material, the modulus of elasticity E, shear modulus G and Poisson's ratio ν are related by

$$G = \frac{E}{2(1+\nu)}$$

$$E = 2G(1+\nu)$$

The bulk modulus (K) describes volumetric elasticity or the tendency of an object's volume to deform when under pressure, it is defined as volumetric stress over volumetric strain, is the inverse of compressibility. The bulk modulus is an extension of Young's modulus to three dimensions.

For an elastic, isotropic material, the modulus of elasticity E, Bulk modulus K, Poisson's ratio ν are related by

$$E = 3K(1 - 2\nu)$$

Note: For an Isotopic materials number of elastic constants are 4 (E, K, G, H)
For Isotropic Materials $E > K > G$
E, K, G are materials properties and always constant for a given material. (They are always non zero +ve constants)

Material	Total Elastic Constant	Independent Elastic Constants

Isotropic	4	2
Orthotropic	12	9
Anisotropic	∞	21

Thermal Stresses

Temperature causes bodies to expand or contract. Change in length due to increase in temperature can be expressed as

$\Delta L = L.\alpha.\Delta t$

Where, L is the length, α (in degree) is the coefficient of linear expansion, Δt (°C) is the temperature change.

From the above equation thermal strain can be expressed as,

$$\epsilon = \frac{\Delta L}{L} = \alpha\Delta t$$

If a temperature deformation is permitted to occur freely no stress will be induced in the structure. But in some cases it is not possible to permit these temperature deformations, which results in creation of internal forces that resist them. The stresses caused by these internal forces are known as thermal stresses.

When the temperature deformation is prevented, thermal stress developed due to temperature change can be given as:

$\sigma = E.\alpha.\Delta t$

Cylindrical Pressure Vessel

Pressure vessels are held together against the gas pressure due to tensile forces within the walls of the container. The normal (tensile) stress in the walls of the container is proportional to the pressure and radius of the vessel and inversely proportional to the thickness of the walls. Therefore pressure vessels are designed to have a thickness proportional to the radius of tank and the pressure of the tank and inversely proportional to the maximum allowed normal stress of the particular material used in the walls of the container.

Spherical Pressure Vessel

Consider a closed thin-walled spherical shell subject to a uniform internal pressure p. The inside radius of the shell is r and its wall thickness is h. This body is acted upon by the applied internal pressure p as well as the forces that the other half of the sphere, which has been removed, exerts upon the half under consideration. Because of the symmetry of loading and deformation, these forces may be represented by axial tensile stresses σ_a as shown in Fig.